

## **Shallow Water Propagation**

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### **LONG-TERM GOALS**

Develop methods for deterministic and stochastic acoustic calculations in complex shallow water environments, specify their capabilities and accuracy, and apply them to explain experimental data and understand physical mechanisms of propagation.

### **OBJECTIVES**

- (A) Treat propagation from narrowband and broadband sources over elastic and poro-elastic sediments, and incorporate realistic bathymetric, topographic, and geoacoustic variations.
- (B) Quantify acoustic interactions with ocean-volume features, including nonlinear internal waves (NIWs) and fronts over variable bathymetry, and analyze and interpret experimental data.
- (C) Characterize effects of sandy and other sediments on propagation, and specify for one class of muds the physical variations that affect geoacoustical properties.

### **APPROACH**

- (A) Develop efficient and accurate parabolic equation (PE) techniques for propagation through heterogeneous sediments. Treat range dependence and sediment layering by single scattering and energy conservation methods. Benchmark results using data and special high-accuracy solutions.

## Report Documentation Page

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- (B) Construct representations for ocean environmental variability using data and parametric models. Determine acoustic fields with PE, normal mode, and other approximation methods. Use experimental data and computational results to assess propagation mechanisms.
- (C) Determine the influence of sediment attenuation on propagation statistics using normal mode methods. Investigate muds for which the current card-house structure theory is applicable. Find sensitivity of geoacoustic property predictions to variability in sediment parameters.
  - Principal collaborators: Rensselaer graduate students, Dr. Michael Collins (NRL), Drs. James Lynch, Ying-Tsong Lin, and Timothy Duda (WHOI), Dr. Allan Pierce (BU, retired), and recent Rensselaer graduates.

## WORK COMPLETED

### (A) Propagation model development

- The seismo-acoustic PE method for range-dependent bathymetry is extended for problems with multiple fluid and solid layers, variable thickness layers, and continuous depth dependence within layers [1], using the single-scattering approximation at vertical interfaces and a newly-applied non-centered difference formula at horizontal fluid-solid interfaces.
- Another important PE method extension is for problems with variable topography [2], for which the single-scattering approximation for vertical interfaces is enhanced by introducing a thin vertical region, in order to account for changes in the location of the top boundary as well as within the interior of the propagation domain.
- An alternative to stair-step approximations for range-dependent bathymetry uses a known coordinate mapping technique, and a generalization is developed [3] that transforms a waveguide with bathymetric and topographic variations into a constant-thickness waveguide, with propagation examples showing high accuracy.
- An application to propagation under ice covers is described in [4], where the PE method is used with a relatively thin elastic layer and stochastically-generated samples of ice keels to illustrate increased transmission loss from both energy absorption by relatively smooth ice and scattering from rough ice.
- Propagation through range-dependent transversely isotropic elastic sediments, which occur in some shallow oceans, is handled by an improved PE solution [5], and examples indicate the circumstances when anisotropic propagation effects can be characterized in terms of “equivalent” isotropic sediments.
- Computational techniques and dependent variables that are designed for range-dependent seismo-acoustic environments are extended to Biot-type poro-elastic sediments [6], and solutions for range-independent examples agree with available benchmark cases and show how to treat problems involving fluid, elastic, and poro-elastic layers.

(B) Ocean variability influence

- Sensitivity of modal properties is determined for variations of parameters in a feature model of a curved front over sloping bathymetry [7], using a series of analytical approximations in the modal dispersion relation, and accurate results require approximations that include the curvature of the wave fronts.
- A scattering model for acoustic energy inside a NIW duct is developed using an adiabatic-mode transport theory, with scattering elements being segments of wave front [8] and a modified diffusion equation describing the evolution of averaged intensity and its dependence on environmental and acoustic parameters.
- The modal transport theory is applied to determine how intersecting NIW fronts influence average intensity [9], which depends strongly on the frontal orientations and on the asymmetry of individual scattering elements.

(C) Sediment geoacoustic properties

- The best available model for card-house platelet interactions accounts for elasticity and end-to-face bonding across a small channel between platelets, and the calculation of an effective shear modulus produces shear speed estimates [10] of approximately the same size as measured for high-porosity marine muds.
- No interaction force occurs between bubbles with surface electric dipoles that have normally-oriented moments, and this result, which generalizes to non-spherical bubbles and weakly off-normal moments [11], suggests implications for the shapes and properties of bubbles in marine mud.
- Porosity estimates of mud satisfying conditions for card-house interactions are found by modeling platelet aggregation as a diffusion-limited cluster-cluster process of fractal type [12], which is validated by finding its fractal dimension and comparing with results determined from laboratory aggregation experiments.
- Using a parametric description of shallow-ocean layered waveguides, convenient approximations for modal attenuation coefficients are determined [13], and their frequency behavior corresponds to that of a class of more realistic waveguides.
- Expressions are derived from adiabatic mode theory for averaged transmission loss in nearly range-independent waveguides in terms of environmental and acoustic parameters [14], and results reduce to well-known results of Rogers at high frequencies for both isospeed and constant-gradient sound speed profiles.

**RESULTS** (from three selected investigations)

(A) An essential capability for acoustic data analysis and for many applications is reliable propagation calculations in shallow water waveguides with range-dependent elastic sediment layers. Elasticity provides an important energy transfer mechanism between compressional and shear wave modes that can substantially modify acoustic intensity and phase. A key physical and computational complication is that energy spectra for elastic media extend over a wider range of wave numbers than for fluids. After decades of research led by Dr. Michael Collins, the goal of a 2-D PE model

for gradually range-dependent elastic sediments that has all needed capabilities and that is accurate, efficient, and robust is nearly at hand. Much progress has been able to be transferred from successes of the fluid PE for shallow water, but critical new components were required to resolve issues for elastic sediments. These include: a robust formulation with appropriate non-traditional dependent variables; improvements in single-scattering treatments at stair-step risers in approximations for range dependence; non-uniform depth gridding to treat thin layers efficiently; and accurate treatment of range-dependent fluid-solid interfaces. One clearly needed capability that was missing until recently is to handle layers with depth-dependent properties (heterogeneous layers). The complication is that at fluid-solid interfaces where either or both of the layers are heterogeneous, one interface condition requires a second derivative. It can be shown that the standard centered-difference formula for a second derivative is not sufficiently accurate. The resolution [1] requires a high-order non-centered-difference formula, one that is rarely seen outside a catalog of such formulas, and it dramatically improves accuracy at no cost in efficiency. As a result, environments with multiple fluid and solid layers, whether homogeneous or heterogeneous, and with range-dependent interfaces can now be handled by the seismo-acoustic PE. A bonus is that the same difference formula provides an effective means for treating variable topography modeled as a solid, such as ice cover and beaches. It turns out for such problems that preserving accuracy requires conceptually splitting a stair-riser interface into two interfaces separated by a very thin region. This split provides sufficient freedom to account for layer heterogeneity and for a change in location of the top boundary [2]. With these and other recent enhancements, propagation calculations for gradual 2-D range-dependent seismo-acoustics problems will reach the same level of capabilities, accuracy, and reliability as for corresponding fluid problems. Future extensions include treating 3-D problems and more complex media, such as transversely isotropic elastic [5] and poro-elastic [6] sediments, and devising new range-dependent seismo-acoustic benchmark examples.

- (B) The importance of fronts and NIWs for 3-D acoustic propagation in shallow ocean regions cannot be overstated, and bathymetric variations may add significant complexity. Results from SW06 and other experiments exhibit phenomena that inspired improved ocean-feature modeling in order to quantify their propagation effects. Consequently, there are research objectives to identify what key parameters in 3-D ocean features should be measured for required accuracy in acoustic quantities, and with what accuracy these parameters must be known. Recent major improvements in computational ocean models now permit tracking the dynamics of such features. Therefore, a related objective is the sensitivity of properties of features in computational predictions that are used for acoustic purposes, as in the Integrated Ocean Dynamics and Acoustics (IODA) project. One approach toward greatly reducing the calculations needed in parameter spaces is to develop relatively simple analytical formulas for interesting acoustic quantities from relevant feature models. A starting point is a 3-D feature model by Drs. Ying-Tsong Lin and James Lynch for an idealized along-shore front that is curved in depth and that lies over sloping bathymetry. Specifically, as range increases from inshore of the constant-curvature front to offshore, the depth increases linearly and the sound speed jumps to a higher value. This environment has two types of acoustic modes that propagate along shore: leaky modes, for which some inshore energy crosses the front; and whispering gallery modes, for which inshore energy remains on that side, so these modes decay much more slowly than leaky modes as they propagate. The initial focus is on modal phase and group velocity formulas, obtained from the exact dispersion relation of the front model, that conveniently and correctly represent the variations with feature parameters. The first lesson learned is that reformulating a complicated dispersion relation may significantly ease the task of approximating it. This led to an initial asymptotic approximation for whispering gallery modes

that effectively neglects frontal curvature and provides a convenient formula for feature parameter dependence. In spite of this dynamically drastic simplification, the formula reproduces well the modal velocity dependence on frontal location, sound speed jump, and acoustic frequency. Its drawbacks are that it applies only for the smallest depth modal number, and it possesses more whispering gallery modes than does the exact expression. These results motivate a set of second approximations which incorporate curvature at the cost of handling asymptotics of Airy functions. Nonetheless, these results too can be further simplified significantly [7], and the formulas not only accurately reproduce the correct parameter dependence but also apply for all depth mode numbers and for the correct distribution of mode types. Future work includes extending the approach to fronts with continuous rather than jump sound speed changes, to straight and curved ducts from NIWs, and to acoustic quantities such as transmission loss and scintillation index.

- (C) Both physical understanding and reasonable estimates of geoacoustic properties for shallow water sediments are essential for scientific and Naval applications. For sandy and silty sediments, experimental and theoretical investigations show that their geoacoustic properties can generally be handled well in propagation calculations for fluid, elastic, or poro-elastic models. Nevertheless for a variety of mud types, particularly high-porosity marine mud (HPMM), a major challenge has been to construct a physically-based model that correctly predicts and explains all the required geoacoustic properties. For example, the compressible sound speed, and only this parameter, can be estimated fairly well using the Mallock-Wood approach for a two-phase medium. In contrast, useful first-principle predictions of HPMM shear sound speed have been unavailable prior to a new paradigm of an aggregated card-house structure, proposed by Dr. Allan Pierce and the late Dr. William Carey. A key feature of the model is that each solid component, which is a thin platelet of clay minerals possessing an electric charge distribution as a result of its chemical, electrical, and material properties, mimics a sheet of distributed longitudinal quadrupoles aligned transversely to the platelet. This structure causes platelets to repel for end-to-end or face-to-face contact and promotes their attraction end-to-face. A recently predicted feature of the attraction is the occurrence of a very narrow channel across which the end-to-face bonding occurs. The attraction leads to formation of card-house structures, probably by a process known as cluster-cluster aggregation that is limited by platelet diffusion. Such an aggregate supports weak shear strains, and its effective shear modulus can be found from its constituent subunits consisting of two interacting platelets. Modeling the electrical and mechanical interactions of platelets in a subunit provides estimates [10] of the effective shear modulus and corresponding shear speed  $c_s$ . One result is that  $c_s$  is sensitive mainly to the platelet's thickness, length, and charge-carrying capacity. For estimates of  $c_s$ , the most common types of minerals in HPMM are kaolinite and smectite, and representative values for the platelet parameters yield corresponding  $c_s$  estimates of 13 m/s and 0.25 m/s. Typically HPMM consists of combinations of these and other minerals, so the effective  $c_s$  would be weighted averages of the estimates. A critical question is, how well do estimates compare with experimental measurements? Recent careful laboratory experiments by Ballard, et al. report  $c_s$  for pure kaolinite with porosity 0.76 as  $6.9 \pm 0.7$  m/s, and the card-house estimate for parameters appropriate to their experiment is 6.1 m/s. While this remarkable agreement is subject to a variety of assumptions and estimates, the close measured and predicted values can reasonably be considered a success for the card-house approach for this mud type. Future work includes improving porosity estimates from the card-house model [12], investigating decreased salt-water bubble coalescence and related bubble phenomena [11],[18], devising physics-based models for platelet interactions in mud [19] where other aggregation processes occur, and finding estimated attenuation.

## IMPACT/APPLICATIONS

New or enhanced capabilities are provided for propagation predictions that depend on physical properties of shallow water sediments, including layering, elasticity, porosity, and anisotropy. Range-dependent variability from bathymetry, topography, and sediment interfaces can be treated in propagation calculations. Intensity attenuation and coherence statistics that result from environmental fluctuations and other experimental variability can be found more efficiently. Data analyses and model comparisons allow specification of the roles of key physical mechanisms, such as linear or nonlinear frequency dependence of sediment attenuation, sediment heterogeneity or homogeneity, water column or bathymetric variability, water column scattering or refraction, and vertical or horizontal mode coupling from nonlinear internal waves and bathymetry. Results from modeling and data analyses of experiments, particularly experiments off the New Jersey Shelf, are partly aimed at improving shallow water sonar systems and predictions. Propagation model implementations, analysis tools, and data representation techniques are distributed to university, laboratory, and other research/development groups.

## RELATED PROJECTS

- Continuing projects with Dr. Michael Collins on increasing PE capabilities and accuracy include [1], [2], and extensions of [6]. In addition a monograph is in process on parabolic equation techniques for scalar and vector acoustic-wave problems and applications [15], for which comprehensive research results are nearly complete.
- Along with investigations with Drs. James Lynch, Y.-T. Lin, and Timothy Duda involving three-dimensional propagation effects in waveguides [8], [9], related work includes curvature influences in internal wave ducts [16] and demonstrating acoustic effects of parameters in ocean feature models [7], [17]. Research under this grant is related to the WHOI-led MURI IODA project.
- Current research with Dr. Allan Pierce focuses on the card-house model of high-porosity marine mud [10], [12] and propagation variability from geoacoustic structure and attenuation [13], [14]. Other projects are concerned with the behavior and structure of bubbles in mud [11], [18] and the physics of platelet interactions [19].

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## PUBLICATIONS

- Published [refereed]: [3], [6], [16]
- Published [non-refereed]: [4], [10]
- Submitted [refereed]: [1], [2], [13]